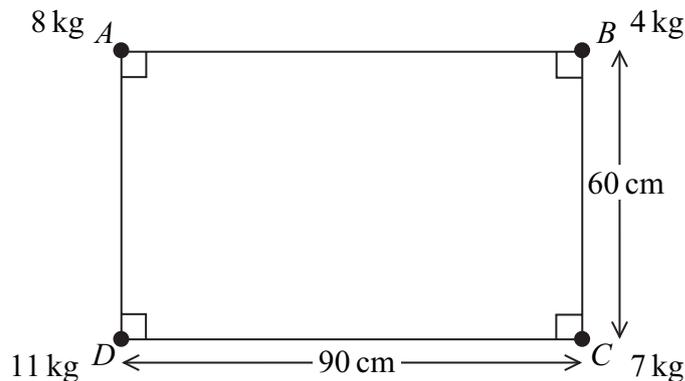AQA Maths M2

Topic Questions from Papers

Centre of Mass

1   Particles of masses 8 kg, 4 kg, 7 kg and 11 kg are attached to the vertices *A*, *B*, *C* and *D* respectively of a light, rigid, rectangular framework *ABCD*.

In the framework, *CD* = 90 cm and *BC* = 60 cm, as shown in the diagram.

(a)   Show that the centre of mass of the system of particles is 33 cm from *AD*.   *(3 marks)*

(b)   Find the distance of the centre of mass of the system of particles from *AB*.   *(3 marks)*

(c)   The framework is freely suspended from the corner *A* and hangs in equilibrium. Find the angle, in degrees, between the downward vertical and the side *AB*.   *(3 marks)*
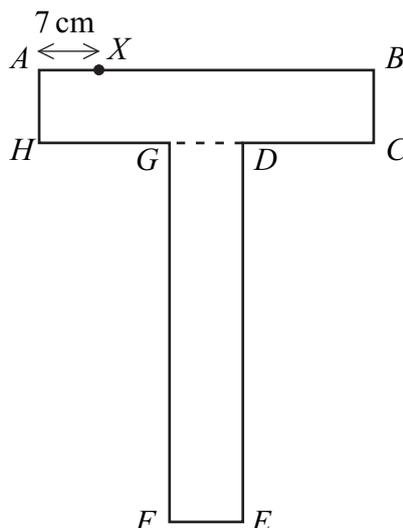
(Q2, June 2006)   *PMT*

2   A uniform T-shaped lamina is formed by rigidly joining two rectangles *ABCH* and *DEFG*, as shown in the diagram.
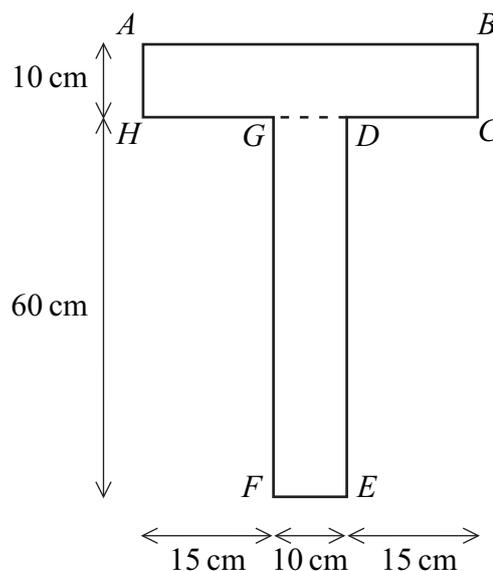
(a)   Show that the centre of mass of the lamina is 26 cm from the edge *AB*.   *(4 marks)*

(b)   Explain why the centre of mass of the lamina is 5 cm from the edge *GF*.   *(1 mark)*

(c) The point X is on the edge AB and is $7\,\text{cm}$ from A, as shown in the diagram below.

7 cm

$A \longleftrightarrow X$ B

H G D C

F E

The lamina is freely suspended from X and hangs in equilibrium.

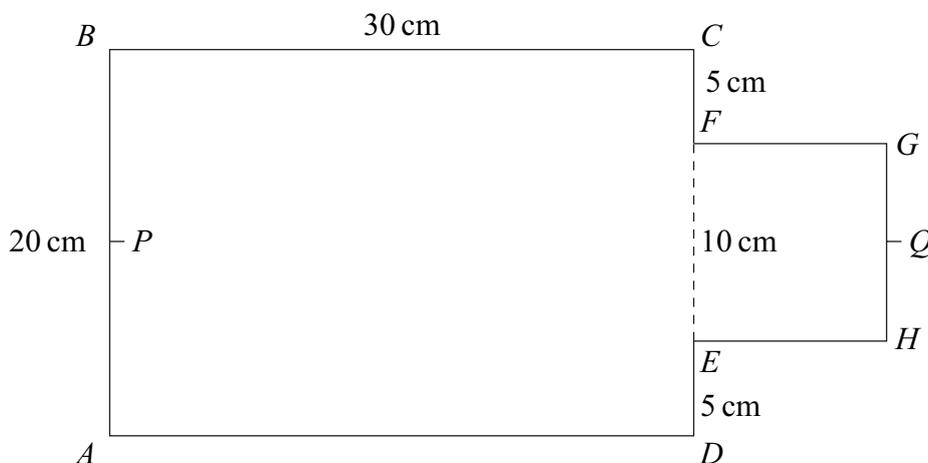Find the angle between the edge AB and the vertical, giving your answer to the nearest degree. *(4 marks)*

(Q2, Jan 2007)

3 A uniform lamina is in the shape of a rectangle $ABCD$ and a square $EFGH$, as shown in the diagram.

The length AB is $20\,\text{cm}$, the length BC is $30\,\text{cm}$, the length DE is $5\,\text{cm}$ and the length EF is $10\,\text{cm}$.

The point P is the midpoint of AB and the point Q is the midpoint of HG.

B 30 cm C

5 cm

F

G

20 cm P 10 cm Q

H

E

5 cm

A D

(a) Explain why the centre of mass of the lamina lies on PQ. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from AB. *(4 marks)*

(c) The lamina is freely suspended from A.

Find, to the nearest degree, the angle between AD and the vertical when the lamina is in equilibrium. *(4 marks)*
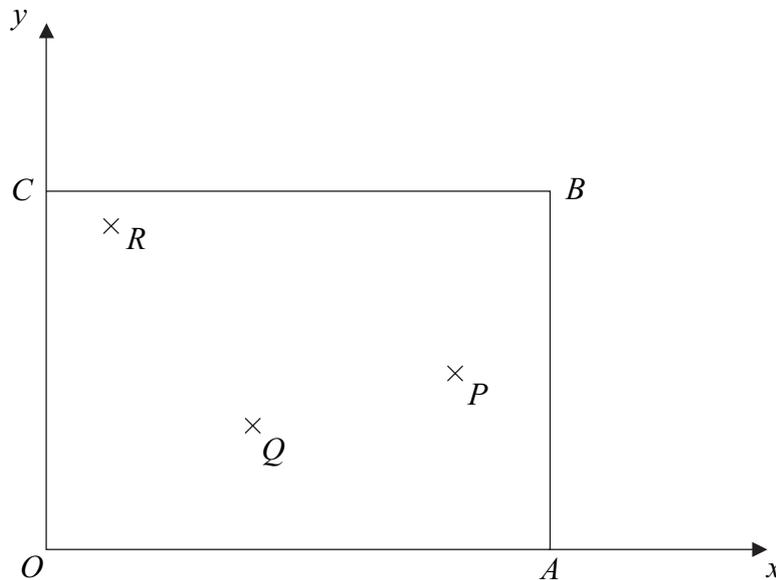
(Q3, June 2007)

4 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

Particle P has mass $1\,\text{kg}$ and is attached at the point $(25, 10)$.
Particle Q has mass $4\,\text{kg}$ and is attached at the point $(12, 7)$.
Particle R has mass $5\,\text{kg}$ and is attached at the point $(4, 18)$.
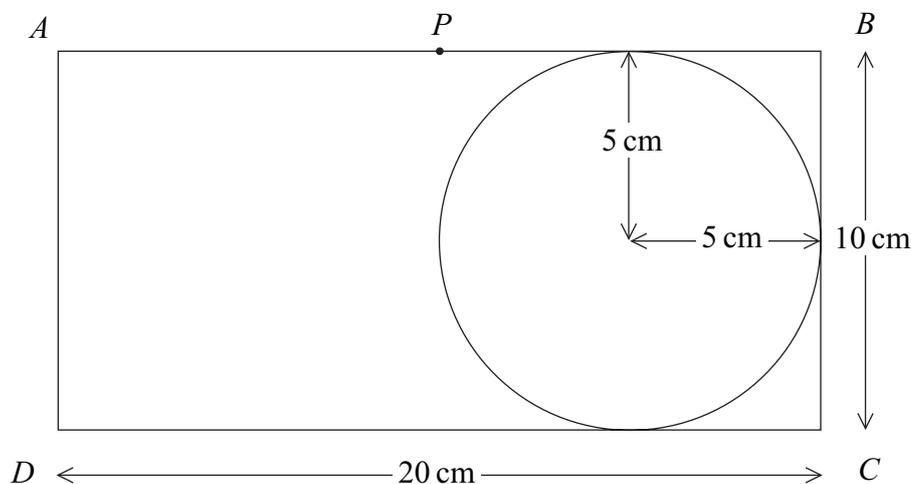

Find the coordinates of the centre of mass of the three particles. *(4 marks)*

(Q3, June 2008)

5 A uniform rectangular lamina *ABCD* has a mass of 8 kg. The side *AB* has length 20 cm, the side *BC* has length 10 cm, and *P* is the mid-point of *AB*.

A uniform circular lamina, of mass 2 kg and radius 5 cm, is fixed to the rectangular lamina to form a sign. The centre of the circular lamina is 5 cm from each of *AB* and *BC*, as shown in the diagram.


(a) Find the distance of the centre of mass of the sign from *AD*.   *(3 marks)*

(b) Write down the distance of the centre of mass of the sign from *AB*.   *(1 mark)*

(c) The sign is freely suspended from *P*.

Find the angle between *AD* and the vertical when the sign is in equilibrium.   *(4 marks)*

(d) Explain how you have used the fact that each lamina is uniform in your solution to this question.   *(1 mark)*

(Q4, Jan 2009)

6 A piece of modern art is modelled as a uniform lamina and three particles. The diagram shows the lamina, the three particles *A*, *B* and *C*, and the *x*- and *y*-axes.
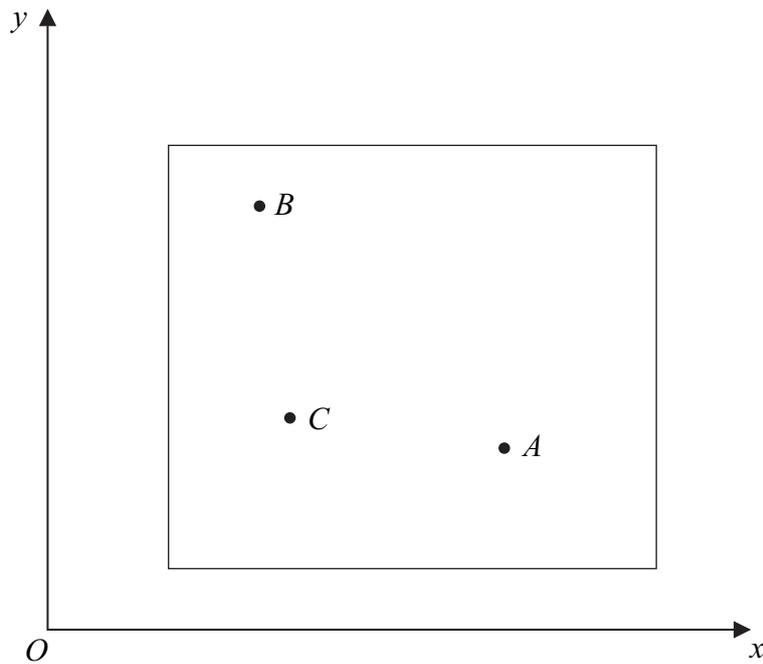
The lamina, which is fixed in the *x*–*y* plane, has mass 10 kg and its centre of mass is at the point (12, 9).

The three particles are attached to the lamina.
Particle *A* has mass 3 kg and is at the point (15, 6).
Particle *B* has mass 1 kg and is at the point (7, 14).
Particle *C* has mass 6 kg and is at the point (8, 7).

Find the coordinates of the centre of mass of the piece of modern art.   *(6 marks)*

(Q2, Jan 2010)

7   A uniform circular lamina, of radius 4 cm and mass 0.4 kg, has a centre *O*, and *AB* is a diameter. To create a medal, a smaller uniform circular lamina, of radius 2 cm and mass 0.1 kg, is attached so that the centre of the smaller lamina is at the point *A*, as shown in the diagram.
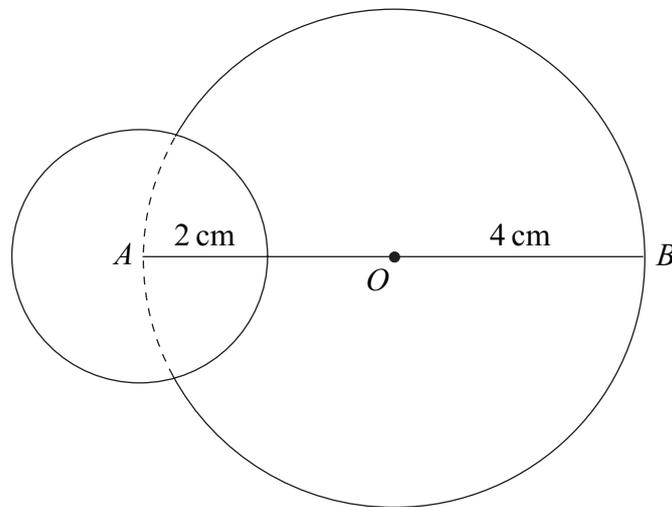

(a)   Explain why the centre of mass of the medal is on the line *AB* .   *(1 mark)*

(b)   Find the distance of the centre of mass of the medal from the point *B*.   *(3 marks)*

(Q3, June 2010)

8   A uniform rectangular lamina *ABCD* has a mass of 5 kg. The side *AB* has length
60 cm and the side *BC* has length 30 cm. The points *P*, *Q*, *R* and *S* are the
mid-points of the sides, as shown in the diagram below.

A uniform triangular lamina *SRD*, of mass 4 kg, is fixed to the rectangular lamina to
form a shop sign. The centre of mass of the triangular lamina *SRD* is 10 cm from
the side *AD* and 5 cm from the side *DC*.
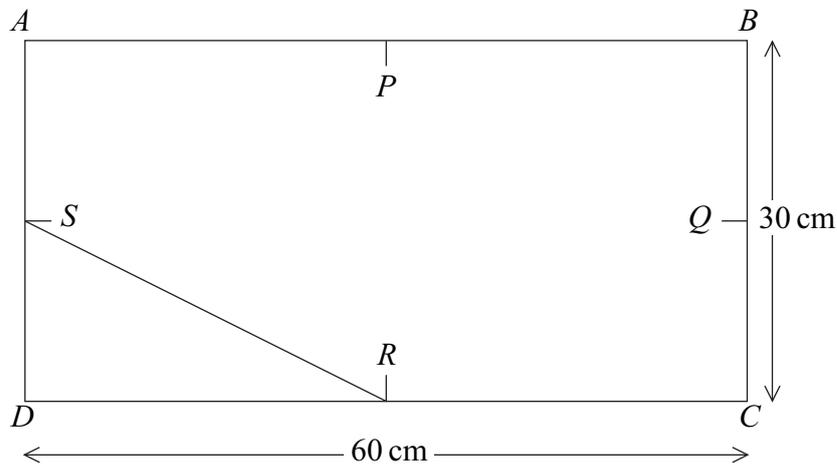
(a)   Find the distance of the centre of mass of the shop sign from *AD*.   *(3 marks)*

(b)   Find the distance of the centre of mass of the shop sign from *AB*.   *(3 marks)*

(c)   The shop sign is freely suspended from *P*.

Find the angle between *AB* and the horizontal when the shop sign is in equilibrium.
(4 marks)

(d)   To ensure that the side *AB* is horizontal when the shop sign is freely suspended from
point *P*, a particle of mass *m* kg is attached to the shop sign at point *B*.

Calculate *m*.   *(3 marks)*

(e)   Explain how you have used the fact that the rectangular lamina *ABCD* is uniform in
your solution to this question.   *(1 mark)*

(Q4, Jan 2011)

9   The diagram shows four particles, *A*, *B*, *C* and *D*, which are fixed in a horizontal plane which contains the *x*- and *y*-axes, as shown.

Particle *A* has mass 2 kg and is attached at the point (9, 6).
Particle *B* has mass 3 kg and is attached at the point (2, 4).
Particle *C* has mass 8 kg and is attached at the point (3, 8).
Particle *D* has mass 7 kg and is attached at the point (6, 11).
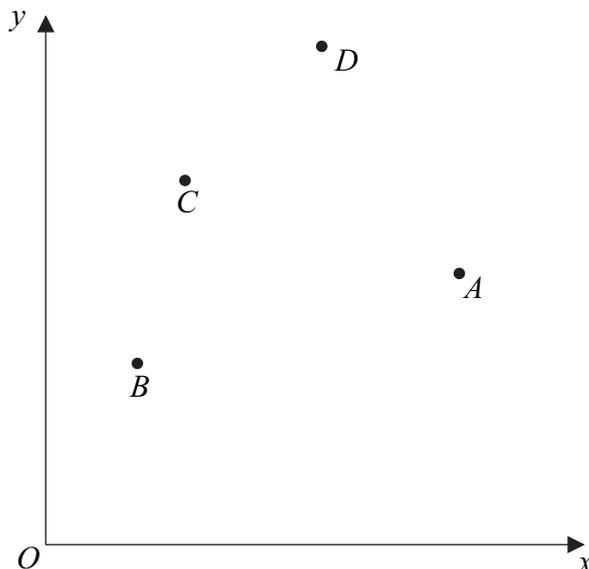
Find the coordinates of the centre of mass of the four particles.   *(5 marks)*

(Q2, June 2011)   PMT

10   A uniform rectangular lamina *ABCD*, of mass 1.6 kg, has side *AB* of length 12 cm and side *BC* of length 8 cm.

To create a logo, a uniform circular lamina, of mass 0.4 kg, is attached. The centre of the circular lamina is at the point *C*, as shown in the diagram.
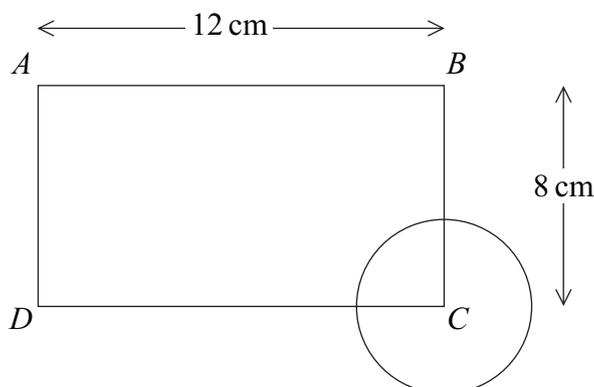
(a)   Find the distance of the centre of mass of the logo:

(i)   from the line *AB*;   *(3 marks)*

(ii)   from the line *AD*.   *(3 marks)*

(b) The logo is suspended in equilibrium, with *AB* horizontal, by two vertical strings. One string is attached at the point *A* and the other string is attached at the point *B*.
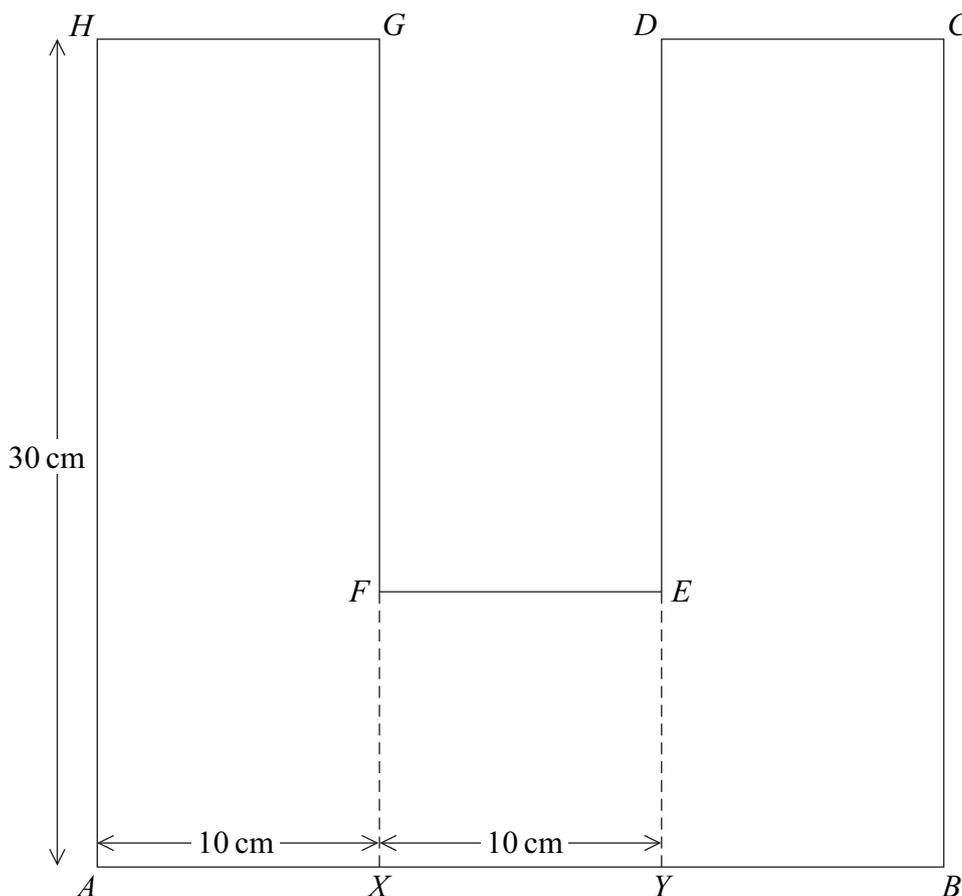
Find the tension in each of the two strings. *(5 marks)*

(Q3, June 2012)   PMT

11 The diagram shows a uniform lamina which is in the shape of two identical rectangles *AXGH* and *YBCD* and a square *XYEF*, arranged as shown.

The length of *AX* is 10 cm, the length of *XY* is 10 cm and the length of *AH* is 30 cm.

(a) Explain why the centre of mass of the lamina is 15 cm from *AH*. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from *AB*. *(3 marks)*

(c) The lamina is freely suspended from the point *H*.

Find, to the nearest degree, the angle between *HG* and the horizontal when the lamina is in equilibrium. *(4 marks)*

(Q4, Jan 2013)